

DERIVATE

A cura di Matteo Fini

matteo.fini@unimi.it

PROPRIETÀ

1. $f(x) \pm g(x) \rightarrow f'(x) \pm g'(x)$

2. $f(x) \cdot g(x) \rightarrow f'(x) \cdot g(x) + f(x) \cdot g'(x)$

3. $\frac{f(x)}{g(x)} \rightarrow \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

DERIVATE ELEMENTARI

funzioni

derivate

$$f(x) = K$$

$$f'(x) = 0$$

$$f(x) = x$$

$$f'(x) = 1$$

$$f(x) = ax$$

$$f'(x) = a$$

$$f(x) = x^k$$

$$f'(x) = kx^{k-1}$$

DERIVATE ELEMENTARI (continuazione)

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x}$$

$$f(x) = a^x$$

$$f'(a^x) = a^x \ln a$$

$$f(x) = \text{sen}(x)$$

$$f'(x) = \cos x$$

$$f(x) = \cos(x)$$

$$f'(x) = -\text{sen}x$$

DERIVATE ELEMENTARI (continuazione)

$$f(x) = \arcsen x \qquad f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$f(x) = \arccos x \qquad f'(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$f(x) = \arctan x \qquad f'(x) = \frac{1}{1+x^2}$$

$$f(x) = \log_a x \qquad f'(x) = \frac{1}{x} \cdot \frac{1}{\ln a}$$

Esempio 1

$$f(x) = e^{-2x}$$

$$f'(x) = -2e^{-2x}$$

Esempio 2

$$f(x) = \sqrt{x} = (x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

Esempio 3

$$f(x) = -3x$$

$$f'(x) = -3$$

DERIVATA DI UNA FUNZIONE COMPOSTA

$$c(x) = g(f(x))$$

$$c'(x) = g'(f(x)) \cdot f'(x)$$

Esempio 1

$$f(x) = \text{sen}(x^2)$$

$$f'(x) = \cos(x^2) \cdot 2x$$

Esempio 2

$$f(x) = \sqrt{4x^2 - 1} = (4x^2 - 1)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} (4x^2 - 1)^{\frac{1}{2}-1} \cdot \overset{\mathbf{4}}{\cancel{8}}x =$$

$$= 4x \cdot (4x^2 - 1)^{-\frac{1}{2}} = \frac{4x}{(4x^2 - 1)^{-\frac{1}{2}}}$$

Esercizi di riepilogo

ESERCIZIO 1

$$f(x) = e^{\sqrt{x + \frac{2x^2}{x+2}}}$$

$$f'(x) = e^{\sqrt{x + \frac{2x^2}{x+2}}} \cdot \left[\frac{1}{2\sqrt{x + \frac{2x^2}{x+2}}} \cdot \left(1 + \frac{4x(x+2) - 2x^2}{(x+2)^2} \right) \right]$$

ESERCIZIO 2

$$f(x) = \log \left[7x^3 + 4e^{4x+9} - \frac{\sqrt{x}}{x+x^3} \right]$$

$$f'(x) = \frac{1}{\left[7x^3 + 4e^{4x+9} - \frac{\sqrt{x}}{x+x^3} \right]} \cdot$$

$$\cdot \left[21x^2 + (4e^{4x+9} \cdot 4) - \frac{\frac{1}{2\sqrt{x}}(x+x^3) - \sqrt{x}(1+3x^2)}{(x+x^3)^2} \right]$$

ESERCIZIO 3

$$f(x) = \frac{e^{[x^2 - 4x^3]}}{\left\{ \frac{1}{2} \log[9x^2 - \log x^2] \right\}}$$

ESERCIZIO 3 (continuazione)

$$f'(x) = \frac{e^{[x^2-4x^3]}(2x-12x^2)\frac{1}{2}\log[9x^2-\log(x^2)]}{\left\{\frac{1}{2}\log[9x^2-\log(x^2)]\right\}^2}$$

$$\frac{-e^{[x^2-4x^3]}\frac{1}{2}\frac{1}{[9x^2-\log(x^2)]}\left(18x-\frac{1}{x^2}2x\right)}{\left\{\frac{1}{2}\log[9x^2-\log(x^2)]\right\}^2}$$

ESERCIZIO 4

$$f(x) = \sqrt{x^2 + 2x - 3}$$

$$\begin{aligned} f'(x) &= \frac{2x+2}{2} \cdot (x^2 + 2x - 3)^{\frac{1}{2}-1} = \\ &= \frac{\cancel{2}(x+1)}{\cancel{2}} \cdot (x^2 + 2x - 3)^{-\frac{1}{2}} = \frac{x+1}{\sqrt{x^2 + 2x - 3}} \end{aligned}$$

ESERCIZIO 5

$$f(x) = \ln\left(\frac{x^2 - 4}{x}\right)$$

$$\begin{aligned} f'(x) &= \frac{1}{x^2 - 4} \cdot \frac{2x \cdot x - (x^2 - 4)}{x^2} = \\ &= \frac{\cancel{x}}{x^2 - 4} \cdot \frac{2x^2 - x^2 + 4}{\cancel{x^2}} = \frac{x^2 + 4}{(x^2 - 4)x} \end{aligned}$$